

Black holes in generalized dilaton gravity in two dimensions

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ABSTRACT

We consider two-dimensional dilaton-gravity theories with a generic exponential potential for the dilaton, and obtain the most general black hole solutions in the Schwarzschild form. We discuss their geometrical and thermodynamical properties. We also study these models from the point of view of gauge theories of the extended Poincaré group and show that they can be considered as gauge theories with broken symmetry. Finally, we examine the theory in a hamiltonian formalism and discuss its quantization and its symmetries.

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1. Introduction

In this paper we study the black hole solutions of a class of two-dimensional gravity-scalar theories with generic power-law scalar potential. As is well-known, in two dimensions the Einstein-Hilbert action is a total derivative and hence cannot be used to construct a 2-dimensional version of general relativity. Only relatively recently it was realized that an action for two-dimensional gravity can be written down if one introduces a scalar field (whose logarithm is called dilaton in string language), which is coupled non-minimally to gravity [1]. Indeed, a full class of lagrangians can be written down, by varying the form of the kinetic and potential terms for the scalar [2]. These can be reduced, via field redefinitions and conformal transformations of the metric to a standard form, with vanishing scalar kinetic term and arbitrary potential [2,3]. In particular, two special cases have been extensively studied: the Jackiw-Teitelboim (JT) theory [1], and the string theory action [4], which correspond to linear and flat potential respectively [5]. In this paper we discuss the more general case of power-law potentials, which interpolate between the two.[†] These potentials are especially interesting because the corresponding actions are equivalent to higher-derivative actions containing powers of the Ricci scalar [6]: higher-derivative theories have indeed been proposed as an alternative way to model gravity in two dimensions [7]. Moreover, the introduction of a potential term in dilaton-gravity theories is useful in the context of string-generated models, since it may provide a mass for the dilaton. With this motivation, a model similar to ours has been studied in four dimensions in [8].

A different formulation of two-dimensional gravity is given by the gauge formalism [9,10]. The JT and string-like models can in fact be written down as topological gauge theories of the anti-de Sitter and extended Poincaré group respectively. It has been shown [6] that also the models considered here can be formulated in the framework of extended Poincaré theory, if some symmetry-breaking terms are added to the gauge-invariant lagrangian. In this paper we proceed further and discuss the quantization of the generalized actions in this framework. We also discuss an alternative formulation of the symmetries of the theory, involving a non-linear generalization of the de Sitter algebra.

The paper is organized as follows: in section 2 we review the model and discuss the physical properties of its exact black hole solutions. In section 3, we discuss the conformally related theory formulated in terms of the metric which is relevant for string theory. In particular, we discuss the black hole solutions in a Schwarzschild gauge, where they exhibit more clearly their physical properties than in the conformal gauge adopted in [6]. In section 4 we reformulate the model in the gauge formalism and comment about its quantization and its invariance properties.

2. The canonical action

We consider the 2-dimensional action:

$$S = \int d^2x \sqrt{g} (\eta R + \Lambda \eta^h) \tag{1}$$

[†] In terms of the dilaton, the potentials are of course exponential.

For $h = 1$, it reduces to that of the JT theory [1], while for $h = 0$, it is conformally related to the low-energy effective string action [4]. It has been shown in [6] that the action (1) is also related to the higher derivative action

$$S = \int d^2x \sqrt{g} R^k \quad (2)$$

by the field redefinition $\eta = kR^{k-1}$, where $h = k/(k-1)$ and $\Lambda = (1-k)k^{-\frac{k}{k-1}}$, $k \neq 0, 1$.

Variation of the action (1) yields the field equations

$$R = -\Lambda h \eta^{h-1} \quad (3a)$$

$$(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^2) \eta + \frac{\Lambda}{2} g_{\mu\nu} \eta^h = 0 \quad (3b)$$

Let us consider the case $\Lambda > 0$. The general static solutions of (3) can be easily found in the Schwarzschild gauge $ds^2 = -Adt^2 + A^{-1}dx^2$ [6,7]: for $h \neq -1$, one has

$$A = \frac{1}{h+1} [(\lambda x)^{h+1} - c] \quad \eta = \lambda x \quad (4)$$

where $\lambda = \sqrt{\Lambda}$ and c is an integration constant. These solutions are related to those found in the conformal gauge in [6], by simply taking the scalar field η as the spacelike coordinate. The curvature of the metric (4) is given by:

$$R = -A'' = -h\lambda^{h+1}x^{h-1} \quad (5)$$

For $c = 0$, the metric is regular (and flat) at $x = \pm\infty$ iff $h < 1$, and is singular at $x = 0$. The opposite happens if $h > 1$. The two special cases $h = 0$ and $h = 1$ correspond respectively to flat and anti-de Sitter space. If $h < 1$, $x = \infty$ is at infinite spatial distance and the singularity at finite distance. If $h > 1$, the singularity at $x = \infty$ is at finite distance, while $x = 0$ is at infinite distance. The two cases present therefore essentially the same physical behaviour, with the rôle of $x = 0$ and $x = \infty$ interchanged. The flat end of the spacetime at spatial infinity is not asymptotically flat in the usual sense, since the metric is not constant there, but is more similar to a horizon. The $c = 0$ configuration can alternatively be written in the unitary gauge as:

$$ds^2 = dr^2 - r^{-2\frac{h+1}{h-1}} dt^2 \quad (6)$$

Let us now consider the case of nonvanishing c . It is easy to see that in general ($h \neq 0, 1$), for $c < 0$ a naked singularity is present at $x = 0$. For $c > 0$, instead, a horizon is placed at $\lambda x = c^{1/(h+1)}$. The metric describes in this case a regular, asymptotically flat black hole spacetime. In the special case $h = 1$, however, the solution describes the constant curvature anti-de Sitter regular black hole discussed in [11]. For $h = 0$, instead, one has flat space in non-standard coordinates.

The ADM mass M of the black hole solutions can be most easily calculated by means of the formalism of Mann [12]: in his notations

$$M = \frac{1}{2\lambda} [\lambda^2 \eta^{h+1} - (\nabla \eta)^2]$$

Substituting the values for the $c > 0$ solutions and subtracting the contribution of the $c = 0$ background, one obtains:

$$M = \frac{\lambda}{2(h+1)} c \quad (7)$$

The temperature of the black hole at the horizon is given by the inverse periodicity of the regular euclidean solution:

$$T = \frac{\lambda}{4\pi} c^{\frac{h}{h+1}} \quad (8)$$

From the formulae above, it is clear that for $h > 0$ or $h < -1$, T is an increasing function of the mass, and vanishes for $M = 0$. The ground state for the Hawking evaporation process is given in these cases by the $c = 0$ solution. For $-1 < h < 0$, instead, the behaviour of T is more similar to that of the Schwarzschild solution in general relativity, with T divergent at $M = 0$ (see fig. 1a). For $h = 0$, T is independent of the mass.

The entropy S can be obtained by integrating the thermodynamical relation $dS = T^{-1} dM$, yielding:

$$S = 2\pi c^{\frac{1}{h+1}} \quad (9)$$

For $h > -1$, the entropy vanishes at $M = 0$ and increases with the mass, while for $h < -1$ it is a decreasing function of M , which diverges for $M = 0$ (see fig. 1b). If $h > 0$ or $h < -1$, the entropy increases when a black hole splits into smaller holes, while it decreases if $-1 < h < 0$. A relation between S and T which is valid for all these models is

$$ST = \frac{\lambda c}{2} = (h+1)M \quad (10)$$

Finally, we consider the special case $h = -1$. The solution of the field equations is in this case:

$$A = \ln(\lambda x) - c \quad \eta = \lambda x \quad (11)$$

Proceeding as before, one can calculate mass, temperature and entropy of the solution. They are given by:

$$M = \frac{\lambda c}{2} \quad T = \frac{\lambda}{4\pi} e^{-c} \quad S = 2\pi e^c \quad (12)$$

The temperature is thus finite for $M = 0$ and decreases for increasing M .

The equations of motion of a particle moving in the metric (4) can be easily obtained by varying the line element. The result is:

$$\frac{dt}{d\tau} = E \sqrt{\frac{h+1}{(\lambda x)^{h+1} - c}} \quad \frac{dx}{d\tau} = \sqrt{E^2 - \frac{\epsilon}{h+1} [(\lambda x)^{h+1} - c]} \quad (13)$$

where τ is the proper time and $\epsilon = 1, 0$ for massive (resp. massless) particles. While for massless particles one simply has $x = E\tau$, massive particles experience a gravitational potential whose shape depends on h (see fig. 2). If $h < 1$ the particles are attracted towards the singularity, while if $h > 1$ they are repelled (we recall that for $h > 1$ the singularity is at infinity). If $h > |1|$, the potential diverges at the singularity as in the Schwarzschild case, while for $h < |1|$ it is regular. The singularity is always reached in a finite proper time, while spatial infinity requires an infinite amount of proper time to be achieved.

To conclude this section, we make some comments about the case of negative Λ . The metric is given in this case by $A = c - \lambda x^{h+1}$, with $\lambda = \sqrt{-\Lambda}$. A cosmological horizon is located at $\lambda x = c^{1/h+1}$, and the solution is regular at the origin iff $h > 1$ (otherwise a naked singularity is present). In this case the solutions are qualitatively similar to the 2-dimensional de Sitter spacetime.

We also notice that it is possible to generalize the solutions (3) to the case of a scalar potential of the form $V(\eta) = \sum_i \Lambda_i \eta^{h_i}$. In this case the solutions are given by

$$A = \sum_i \frac{\Lambda_i}{h_i + 1} x^{h_i+1} - c \quad \eta = x$$

These solutions are of course more complicated than (3): in particular, they may possess multiple horizons.

3. The stringy action

A conformal transformation of the metric $\hat{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$, with $e^{-2\phi} = \eta$, puts the action (1) in the form [6]:

$$S = \int d^2x \sqrt{\hat{g}} e^{-2\phi} [\hat{R} + 4(\hat{\nabla}\phi)^2 + \Lambda e^{-2h\phi}] \quad (14)$$

This is a generalization of the low-energy string action of [4], which is obtained in the limit $h = 0$. The field equations

$$4\hat{\nabla}^2\phi - 4(\hat{\nabla}\phi)^2 + (1+h)\Lambda e^{-2h\phi} + \hat{R} = 0 \quad (15a)$$

$$-2\hat{\nabla}_\mu \hat{\nabla}_\nu \phi - \hat{g}_{\mu\nu} [-2\hat{\nabla}^2\phi + 2(\hat{\nabla}\phi)^2 - \frac{\Lambda}{2} e^{-2h\phi}] = 0 \quad (15b)$$

are solved in the Schwarzschild gauge $ds^2 = -Adt^2 + A^{-1}dx^2$, by ($\lambda^2 = \Lambda > 0$):

$$\hat{A} = \frac{1}{h+1} (e^{h\lambda y} - ce^{-\lambda y}) \quad \hat{\phi} = -\frac{\lambda}{2} y \quad (16)$$

These solutions can be obtained from those discussed in the conformal gauge in [6], by choosing the dilaton as the spatial coordinate. Alternatively, they can be obtained from

(4), performing the conformal transformations $\hat{g}_{\mu\nu} = e^{2\phi}g_{\mu\nu}$ and then defining the new variable $\lambda y = \ln(\lambda x)$. The curvature of the solution is given by

$$\hat{R} = \frac{\lambda^2}{h+1}(h^2 e^{h\lambda y} - c e^{-\lambda y}) \quad (17)$$

If $c = 0$, the metric is regular at $y = -\infty$ and singular at $y = \infty$ for positive h (the opposite for negative h). For $h = 0$, one has flat space. The singularity is at finite spatial distance, while the regular spacetime end is at infinite distance. The metric is flat at infinity, but not asymptotically flat in the usual sense, since $y = \infty$ is a null line. In the unitary gauge, the $c = 0$ solutions can also be written for nonvanishing h as

$$ds^2 = d\rho^2 - \rho^{-2} dt^2 \quad (18)$$

If $c \neq 0$ one must distinguish two cases: for positive h the metric is singular at both $y = \pm\infty$. If instead $h < 0$, the metric is regular at $y = \infty$ and diverges at $y = -\infty$. Moreover, a horizon is present at $e^{(h+1)\lambda y} = c$, for positive c . In this case, the metric describes a black hole with singularity at $y = -\infty$ and asymptotically flat at positive infinity. If $c < 0$, instead, a naked singularity is present at finite distance. For $h = 0$, one recovers the MSW solutions of the string lagrangian [4].

The mass of the black hole solutions can be obtained as before, following [12]: in the present case

$$M = \frac{1}{2\lambda}[\lambda^2 e^{-2(h+1)\phi} - 4e^{-2\phi}(\nabla\phi)^2]$$

Subtracting the background value ($c = 0$), one has:

$$M = \frac{\lambda}{2(h+1)}c \quad (19)$$

which coincides with (7). Analogously, for the temperature and the entropy of the solutions, one obtains the results (8) and (9).

In the special case $h = -1$, the solution is given by

$$A = (\lambda y - c)e^{-\lambda y} \quad \phi = -\frac{\lambda}{2}y \quad (20)$$

Again, its thermodynamical parameters coincide with those of the corresponding solution (11), which are listed in (12).

The motion of a particle in the metrics (16) is obtained by solving the differential equations:

$$\frac{dt}{d\tau} = E \sqrt{\frac{h+1}{e^{h\lambda y} - c e^{-\lambda y}}} \quad \frac{dy}{d\tau} = \sqrt{E^2 - \frac{\epsilon}{h+1}(e^{h\lambda y} - c e^{-\lambda y})} \quad (21)$$

where E is the energy of the particle and τ its proper time. Again, massless particles move along $y = E\tau$, while massive particles experience the potential depicted in fig. 3.

For $h < 0$, the potential is attractive at short distances and repulsive at large distances. A particle coming from infinity must therefore have energy $E^2 > (-c/h)^{h/(h+1)}$ in order to reach the singularity. For positive h , instead, the particle is always repelled from the singularity (placed at $y = \infty$).

The solutions with $\Lambda < 0$ are given by $A = ce^{-\lambda y} - e^{h\lambda y}$, $\lambda = \sqrt{-\Lambda}$. They have a cosmological horizon at $e^{(h+1)\lambda y} = c$, but all are singular at $y = -\infty$ and hence do not seem to be physically relevant.

Also in this case the solutions (16) can be generalized to potentials of the form $V(\phi) = \sum_i \Lambda_i e^{-2h_i \phi}$. The solutions are given now by:

$$A = \sum_i \frac{\Lambda_i}{h_i + 1} e^{h_i x} - ce^{-x} \quad \phi = -\frac{x}{2}$$

The properties of these solutions depend of course on the specific form of the coefficients Λ_i and h_i .

4. The gauge formulation

In some circumstances, two-dimensional dilaton-gravity models can be considered as gauge theories. In particular, it has been shown [9], that the JT model can be formulated as a gauge theory of the 2-dimensional (anti)-de Sitter group. Similarly, the string-inspired model ($h = 0$), can be thought of as a gauge theory of the 2-dimensional extended Poincaré group [10]. In this section we show that also our generalized models can be formulated as gauge theories of the extended Poincaré group, if a symmetry-breaking term is added to the lagrangian [6].

Consider the 2-dimensional extended Poincaré algebra [10]:

$$[P_a, J] = \epsilon^a_b P_b \quad [P_a, P_b] = \epsilon_{ab} I \quad [P_a, I] = [J, I] = 0$$

and the corresponding gauge field:

$$A = e^a P_a + \omega J + a I \quad (22)$$

where e^a is the zweibein and ω is the spin connection. The field strength is given by

$$F = dA + A^2 = P_a T^a + J d\omega + I \left(da + \frac{1}{2} \epsilon_{ab} e^a e^b \right) \quad (23)$$

with the torsion $T^a \equiv de^a + \epsilon^a_b \omega e^b$.

According to [8], the fields transform under the gauge transformations generated by $\Theta = \theta^a P_a + \alpha J + \beta I$, as:

$$\begin{aligned} e^a &\rightarrow (\mathcal{M}^{-1})^a_b (e^b + \epsilon^b_c \theta^c + d\theta^b) \\ \omega &\rightarrow \omega + d\alpha \\ a &\rightarrow a - \theta^a \epsilon_{ab} e^b - \frac{1}{2} \theta^2 \omega + d\beta + \frac{1}{2} d\theta^a \epsilon_{ab} \theta^b \end{aligned} \quad (24)$$

where $\mathcal{M}_b^a = \delta_b^a \cosh \alpha + \epsilon_b^a \sinh \alpha$.

One can now define the gauge multiplet of scalar fields $\eta_A = (\eta_a, \eta_2, \eta_3)$, which permits to construct the topological lagrangian:

$$\frac{1}{2}\mathcal{L}_1 = \Sigma \eta_A F^A = \eta_a T^a + \eta_2 d\omega + \eta_3 (da + \frac{1}{2}\epsilon_{ab} e^a e^b) \quad (25)$$

invariant under the extended Poincaré group. To this we add a symmetry-breaking term, which is invariant only with respect to the subgroup $U(1) \times U(1)$, generated by J and I :

$$\frac{1}{2}\mathcal{L}_2 = \chi_1 da + \chi_2 \left(\eta_3 - \frac{\Lambda}{2} \eta_2^h \right) \quad (26)$$

The lagrangian multipliers χ_i enforce the constraints:

$$da = 0 \quad \eta_3 = \frac{\Lambda}{2} \eta_2^h \quad (27)$$

Solving these constraints, one obtains the lagrangian

$$\frac{1}{2}\mathcal{L} = \eta_a T^a + \eta_2 d\omega + \frac{\Lambda}{4} \eta_2^h \epsilon_{ab} e^a e^b \quad (28)$$

For $h = 1$, the action (28) coincides with that of anti-de Sitter gravity [9]. For $h = 0$, instead, the constraints (27) reproduce the explicit solution [10] for a and η_3 of the field equations of the unconstrained lagrangian (25).

The field equation stemming from (28) are

$$\begin{aligned} T^a &= 0 \\ R + h\Lambda\eta_2^{h-1} &= 0 \\ d\eta_a + \epsilon^b_a \omega \eta_b + \frac{\Lambda}{2} \eta_2^h \epsilon_{ab} e^b &= 0 \\ d\eta_2 + \eta_a \epsilon^a_b e^b &= 0 \end{aligned} \quad (29)$$

The first equation implies the vanishing of the torsion and hence the usual relation between the spin connection and the zweibein, while the second coincides with (3a), provided one identifies η_2 with η . Finally, the last two equation, combined, yield (3b). The solutions of the field equations are therefore given by (4), and hence the nonvanishing components of the zweibein and the connection are:

$$e_t^0 = \sqrt{\frac{(\lambda x)^{h+1} - c}{h+1}} = (e_x^1)^{-1} \quad (30)$$

$$\omega_t = \frac{1}{2} \lambda^{h+1} x^h \quad \omega_x = 0$$

Moreover, η_2 can be identified with the scalar field η in (1), yielding

$$\eta_2 = \lambda x \quad (31)$$

Finally, one can obtain η_a by solving the third equation (29). The result is:

$$\eta_0 = -\lambda \sqrt{\frac{(\lambda x)^{h+1} - c}{h+1}} \quad \eta_1 = 0 \quad (32)$$

One can now evaluate the mass of the solution as in [13]; after subtracting as usual the vacuum contribution, one gets:

$$M = \eta_a e_t^a + \eta_2 \omega_t \Big|^\infty = \frac{\lambda}{2(h+1)} c \quad (33)$$

which is the same result (7) obtained before in the geometric approach.

In order to quantize the model, it is interesting to consider the hamiltonian formulation of the equations of motion. The lagrangian (27) can be written, after integration by parts:

$$\frac{1}{2} \mathcal{L} = \eta_a \dot{e}_1^a + \eta_2 \dot{\omega}_1 + e_0^a (\eta'_a + \epsilon_a^b \eta_b \omega_1 + \frac{\Lambda}{2} \eta_2^h \epsilon_{ab} e_1^b) + \omega_0 (\eta'_2 + \eta_a \epsilon_a^b e_1^b) \quad (34)$$

where a dot denotes time derivative and a prime spatial derivative.

The lagrangian (34) has a canonical structure, with coordinates (e_x^a, ω_x) , conjugate momenta (η_a, η_2) and Lagrange multipliers (e_t^a, ω_t) enforcing the constraints:

$$\begin{aligned} G_a &= \eta'_a + \epsilon_a^b \eta_b \omega_x + \frac{\Lambda}{2} \eta_2^h \epsilon_{ab} e_x^b = 0 \\ G_2 &= \eta'_2 + \eta_a \epsilon_{ab} e_x^b = 0 \end{aligned} \quad (35)$$

which imply the conservation of the quantity $\eta^a \eta_a - \frac{\Lambda}{h+1} \eta_2^{h+1}$, whose value coincides with the mass of the solution. The algebra of constraints is given by

$$\{G_a, G_2\} = \epsilon_a^b G_b \quad \{G_a, G_b\} = \epsilon_{ab} \frac{\Lambda}{2} (\eta_2^h)' G_2 \quad (36)$$

where the structure constants are functions of the fields. The algebra looks like a deformation of the SO(2,1) anti-de Sitter algebra and corresponds to a non-linear local symmetry of the action (28), generated by the infinitesimal transformations:

$$\begin{aligned} \delta e^a &= d\xi^a + \epsilon_a^b (\xi^b \omega - \xi^2 e^b) & \delta \omega &= d\xi^2 - \frac{\Lambda}{2} h \eta_2^{h+1} \epsilon_{ab} \xi^a e^b \\ \delta \eta_a &= \epsilon_a^b \left(\frac{\Lambda}{2} \xi_b \eta_2^h + \xi^2 \eta_b \right) & \delta \eta_2 &= \epsilon_a^b \xi^a \eta_b \end{aligned}$$

The quantization can now be straightforwardly performed by replacing the Poisson brackets with commutators and imposing the Gauss law on the physical states, with an appropriate operator ordering. In a Schrödinger representation $e^a \rightarrow i \frac{d}{d\eta_a}$, $\omega \rightarrow i \frac{d}{d\eta_2}$, the constraint equations become:

$$\left(\eta'_a + i \epsilon_a{}^b \eta_b \frac{\partial}{\partial \eta_2} + i \frac{\Lambda}{4} \epsilon_{ab} \eta_2^b \frac{\partial}{\partial \eta_b} \right) \Psi(\eta_a, \eta_2) = 0 \quad (37a)$$

$$\left(\eta'_2 + i \epsilon^a{}_b \eta_a \frac{\partial}{\partial \eta_b} \right) \Psi(\eta_a, \eta_2) = 0 \quad (37b)$$

The solution of (37) is analogous to that given in [14] for the $h = 1$ case:

$$\Psi = \delta \left(\left[\eta^a \eta_a - \frac{\Lambda}{h+1} \eta_2^{h+1} \right]' \right) e^{i\Omega} \psi_\Lambda(M) \quad (38)$$

where M is the mass (7) of the solution and

$$\Omega = \int dx \frac{\eta_2 \epsilon^{ab} \eta_a \eta'_b}{\eta^c \eta_c} \quad (39)$$

The physical states are therefore classified by the mass M . We shall not consider further the quantization of the model, which has also been studied in the geometrical formulation in [15].

5. Final remarks

We have studied the geometrical and thermodynamical properties of two-dimensional gravity-scalar black holes in two different gauges corresponding to the canonical and string metric. They possess identical thermodynamical properties, but have different geometries. Which of the two gauges is relevant for physics depends of course on the specific form of the matter coupling, which has not been discussed here.

We also have investigated the gauge formulation of the theory. Our models can be obtained by adding symmetry-breaking terms to the extended Poincaré group theory. This symmetry breaking could also be obtained dynamically: preliminary results indicate that it can indeed be obtained at the expense of introducing non-vanishing torsion into the theory.

Alternatively, we have shown that the action of the theory is invariant under a non-linear deformation of the anti-de Sitter group, with field-dependent structure constants. It would be interesting to investigate in more detail the mathematical structure of this symmetry.

Acknowledgements

I wish to thank the Departament de Física i Enginyeria Nuclear of the Universitat Politècnica de Catalunya and especially Ll. Ametller for kind hospitality during the first stages of the preparation of this work.

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